1. a) True
b) False
c) False
d) False
e) True
f) True
g) False
h) True
i) True
j) False
[1 mark for each correct part]
2. a) $T(n)=\Theta\left(n^{\frac{1}{2}} \lg n\right)$
b) $T(n)=\Theta\left(n^{3}\right)$
c) $T(n)=\Theta(n)$
d) $T(n)=\Theta\left(n^{2} \lg n\right)$
e) $T(n)=\Theta\left(n^{2}\right)$
[2 marks for each part]
3. 

a) Comparison sort requires $\Omega(n \lg n)$ operations in the worst case.
b) Examples include heap sort, merge sort and introsort.
c) i. Array B. ii. $\mathrm{C}[\mathrm{i}]$ is the number of elements in the input less than or equal to i .
d)

RADIX-Sort $(A, d)$
1 for $i=1$ to $d$
2 use a stable sort to sort array $A$ on digit $i$
e) Because otherwise, the sorting achieved on each iteration could be lost on the next iteration.
[2 marks for each correct part]
4.
a) $O(n)$
b) $O$ (1)
c) $O(1)$
d) $O(n)$
e) $O(1)$
[2 marks for each correct part]
5.
a) $\backslash$ Theta( $\lg n$ )
b) 255678 (or "nothing, because there is no return statement" - we have to allow this alternative answer because the question asks what the algorithm "returns" not what it "prints out")
c) $\Theta(n)$
d) $\Theta(n)$
e) $\Theta(\lg n)$
[2 marks for each correct part]
6.
(e)

(f)

(g)

(h)

(i)

[10 marks, 2 for each iteration]
7.
a)

b)

c) $O(E+V)$
d) $O\left(V^{2}\right)$
e) Adjacency list, because if E is low, then $\mathrm{E}+\mathrm{V}$ is much less than $\mathrm{V}^{2}$.
[2 marks for each correct part]
8.
a)

Stack-Empty $(S)$
1 if S.top $==0$
2 return TRUE
3 else return FALSE
or just
Stack-Empty(S)
return S.top $=0$
[3 marks]
b)
$\operatorname{Push}(S, x)$
1 S.top $=$ S.top +1
$2 S[$ S.top $]=x$
c)

Pop ( $S$ )
if Stack-Empty $(S)$
error "underflow"
else S.top $=$ S.top -1
4 return $S[S$. top +1$]$ [4 marks]
9.
a) If the keys are independently and uniformly distributed across the interval $[0,1)$.
b) $h(k)=k \bmod m$
c) Because if $m=2^{p}$ then $\mathrm{h}(\mathrm{k})$ is just the $p$ lowest-order bits of $k$. Unless we know that all low-order $p$-bit patterns of $k$ are equally likely, then it is better to make the hash function depend on all the bits of $k$.
d) A prime number not too close to a power of 2 but fairly close to $n / \alpha$ where $\alpha$ is the load factor and $n$ is the number of elements to be stored.
e)

We can design a universal class of hash functions as follows:

1. Choose a prime number $p$ greater than the largest possible value of $k$. Let $\mathbb{Z}_{p}=\{0, \ldots p-1\}$ and $\mathbb{Z}_{p}^{*}=\{1, \ldots p-1\}$. We assume $p>m$, since $p$ is greater than the maximum value of $k$
2. We then define the universal class of hash functions

$$
\mathcal{H}_{p m}=\left\{h_{a b}: a \in \mathbb{Z}_{p}^{*} \text { and } b \in \mathbb{Z}_{p}\right\}
$$

where

$$
h_{a b}(k)=((a k+b) \bmod p) \bmod m
$$

[2 marks for each correct part]

10
a) Liskov substitution principle states that if $p$ is a variable that refers to an object of class $x$, then $p$ may refer to any object from any subclass of $x$.


For example, in the above diagram, if $p$ refers to an object of class Shape, then it can also refer to any object whose class is Circle, Triangle or Square.
[4 marks]
b) Observer pattern [2 marks]
c) CompositeView should be a subclass of View. Example of the Composite pattern. [4 marks]

